# Calculus 3

Math = Fun

Calculus - study of FUNctions

Calculus 3 - 3x as much fun.

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Calculus in multiple dimensions

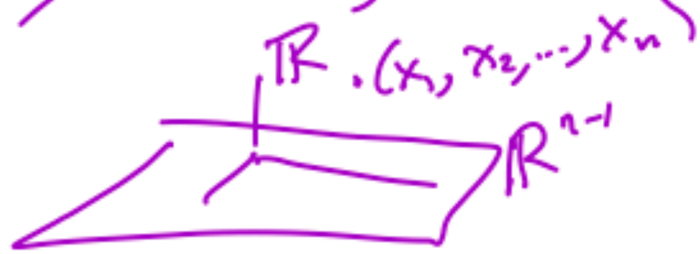
points in $\mathbb{R}^n$ = n-dimensional space

$\mathbb{R}^1 = \mathbb{R}$

$\mathbb{R}^2$ = plane


$\mathbb{R}^3$ =


$\mathbb{R}^n$ =



Good news — we can understand higher dimensions as $(\_, \_, \_, \dots)$ tuples of numbers. ← Can algebraically manipulate these. — Don't necessary have to be able to imagine it. But sometimes it does help if we can imagine it in lower dimensional situations.

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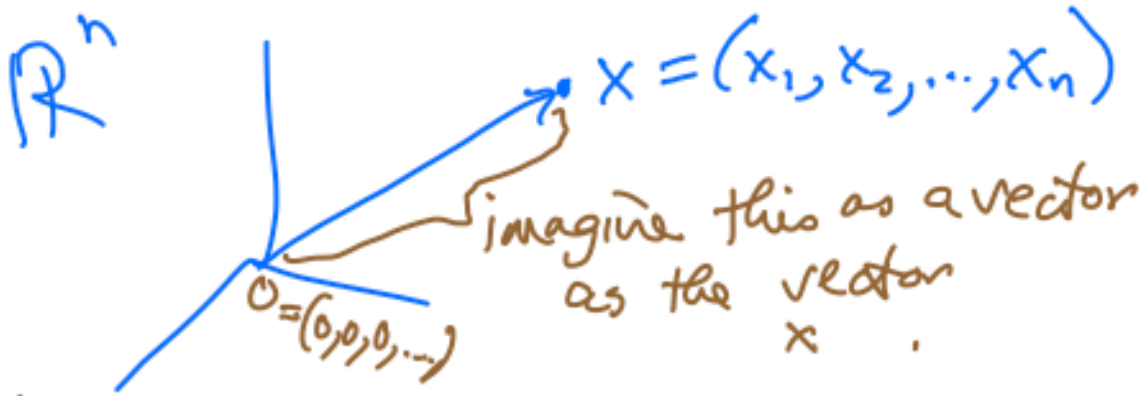
<u>Vectors</u> — think of as an arrow.
↑ really the same as points in $\mathbb{R}^n$




The vector $x$ starts at the origin $0 = (0,0,\dots)$ and the head of the arrow is at $x = (x_1, x_2, \dots)$.


In some books, they will designate when we want to think of $(x_1, x_2, \dots)$ as a vector by writing $\vec{x}$.

Another notation — 2- and 3-d

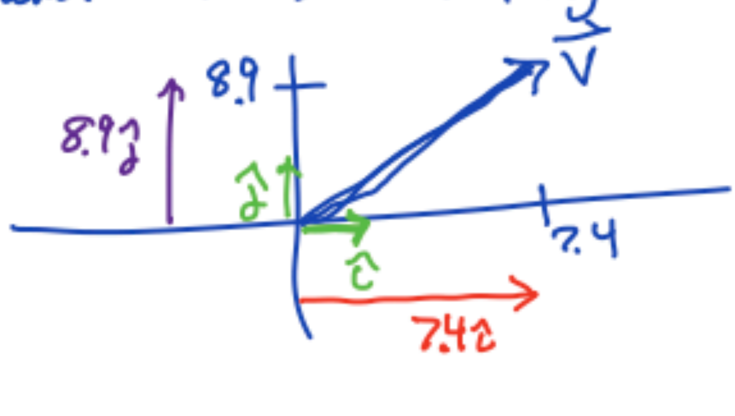
We write $\vec{v} = 7.4\hat{\imath} + 8.9\hat{\jmath}$

means $\vec{v} = (7.4, 8.9)$

We think $\hat{\imath} = (1, 0) =$ unit vector in $x$ direction

$\hat{\jmath} = (0, 1) =$ unit vector in $y$ direction.

"unit" means the length is 1.




$7.4\hat{\imath}$

$= 7.4(1, 0)$

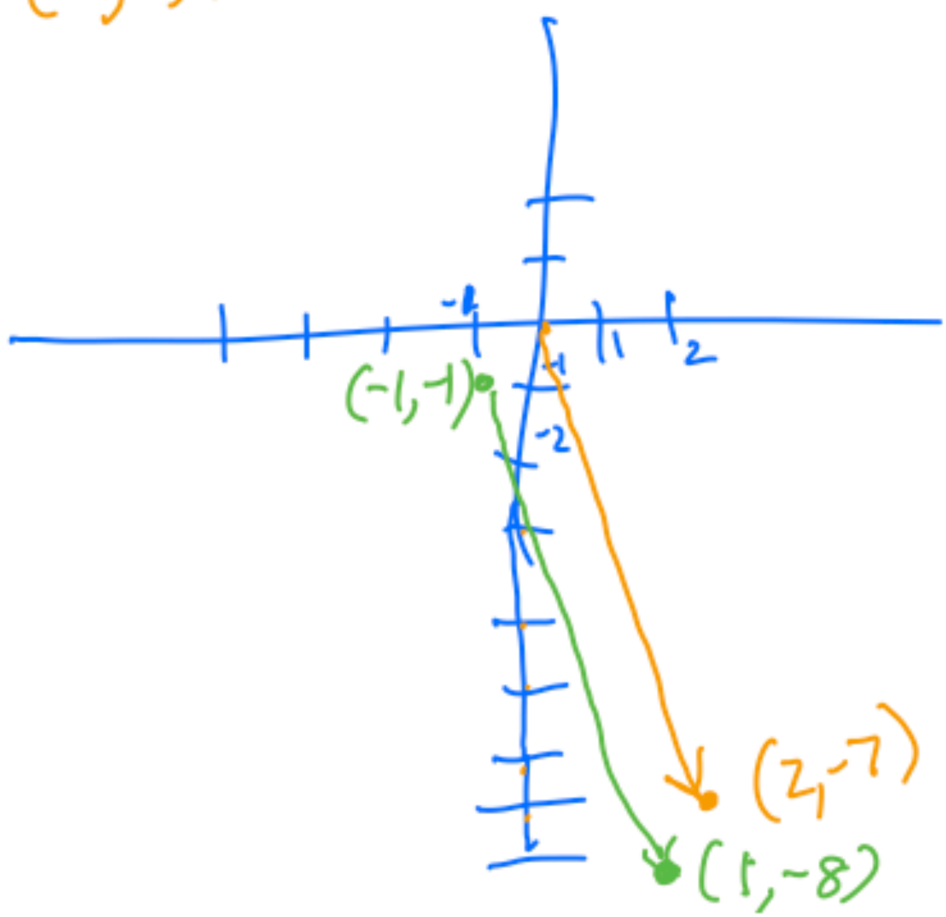
$= (7.4, 0)$

Visualizing arrow vectors —
we are allowed to start at different points besides O. ← we must parallel translate the vector

↑ move without changing the length or direction

eg $\vec{w} = (2, -7)$

⟿ move so it starts at $(-1,-1)$.



Two vectors are the same if the start at different points if the coordinates $(\Delta x, \Delta y)$ are the same

$$(x_2 - x_1, y_2 - y_1)$$


$$(x_2 - x_1, y_2 - y_1) = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}.$$

3-d version

$$(x_1, x_2, x_3) = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$$
$$= \langle x_1, x_2, x_3 \rangle$$

another common notation.

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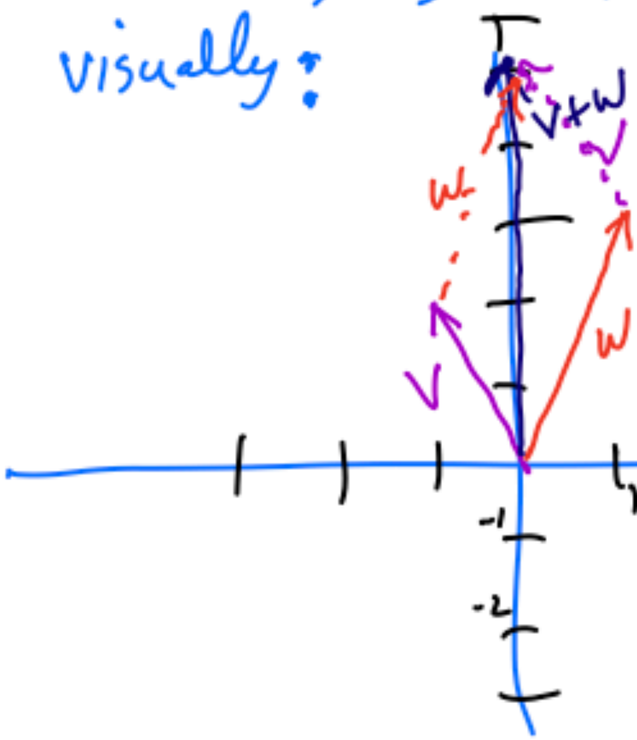
Algebraic operations in $\mathbb{R}^n$ (with vectors)

• Addition & subtraction.

$$(1,2,3.8,-4) - (-1,3,4,8)$$
$$= (2,-1,-0.2,-12).$$

visually:




$V = (-.95, 2)$

$W = (1.04, 3.1)$

$V + W = (.09, 5.1)$

$= W + V$

We can imagine vector addition as putting the vectors head-to-tail.

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Picture of subtracting vectors — leave vectors with same starting point, connected the heads

(so that it looks right for an addition)

In higher dimensions, the picture is the same, with the operation confined to the plane containing the two vectors.

(2 points determine a line
3 points determine a plane)

1 vector determines a line

2 vectors determines a plane.

but note we can move vectors by parallel transport.

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## Scalar Multiplication

real # or complex # (as opposed to a vector)

If $r \in \mathbb{R}$ ← "the set of all real numbers"

∈ ← "is an element of"

(i.e. $r$ is a scalar) and $v \in \mathbb{R}^n$

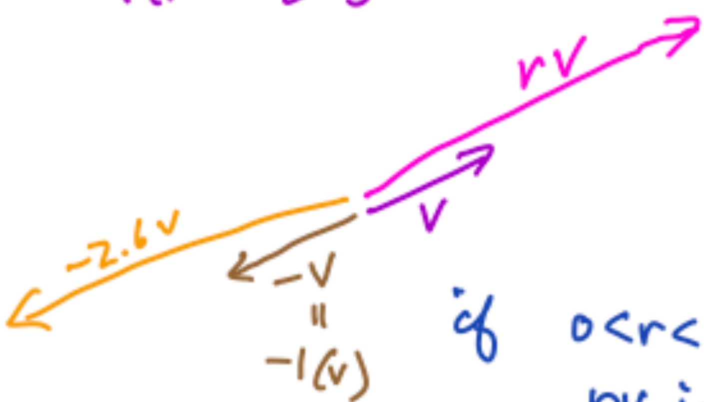
with $v = (v_1, v_2, \dots, v_n)$

Then $rv$ is defined to be

$$rv = (rv_1, rv_2, \ldots, rv_n)$$

eg $-7(1,6,2) = (-7,-42,-14)$.

Visualizing Scalar Mult:




if $r > 1$, same direction with length multiplied by $r$.

if $0 < r < 1$, then $rv$ is shorter than $v$.

For every $r \in \mathbb{R}$, $rv$ is parallel to $v$.

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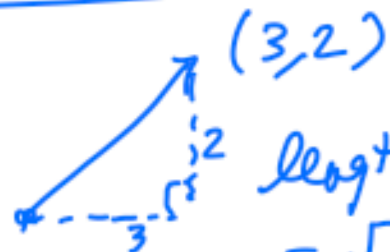
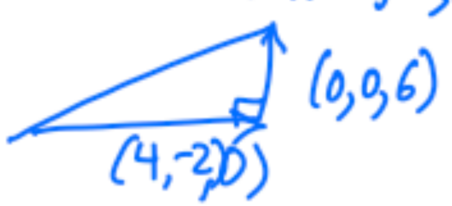



length of $(3,2)$

$= \sqrt{3^2 + 2^2} = |(3,2)|$

$= \|(3,2)\|$

$|(4,-2,6)| = \sqrt{\|(4,-2,0)\|^2 + \|(0,0,6)\|^2}$



$$= \sqrt{4^2 + (-2)^2 + 6^2}$$

in $\mathbb{R}^n$, $\forall x \in \mathbb{R}^n$ ("for all $x$ in $\mathbb{R}^n$")

$$|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

mega-Pythagorean theorem.